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For Susan

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Preliminary Thoughts on a Rational Constructivist Approach to Cognitive Development PRIMITIVES, SYMBOLS, LEARNING, AND THINKING

Fei Xu

INTRODUCTION

This chapter considers a newly emerging view of cognitive development: rational constructivism. I will attempt to sketch the view as I see it, in broad strokes. I will draw on arguments and evidence to see if an overall picture will emerge. Two key developmental issues are discussed: how to characterize the initial state, and how to characterize mechanisms of learning and developmental change. I will argue for the following theses: (1) Infants are much smarter and much more sophisticated learners than what William James, Piaget, or Quine had thought; infants' world is not "a blooming, buzzing confusion." However, it remains unclear how best to characterize the initial state. Instead of sensorimotor primitives or core knowledge, the initial state may perhaps be best characterized as a set of proto-conceptual primitives. (2) Over the last several decades of research on cognitive development three types of learning mechanisms have been uncovered: language and symbol learning as a vehicle for conceptual development; Bayesian learning as a tool for belief revision; and explanation, analogy, and related processes as ways to organize factual knowledge and generate new hypotheses that drive genuine conceptual change. These mechanisms may be considered both rational and constructive.

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THE INITIAL STATE OF A HUMAN INFANT: PROTO-CONCEPTUAL PRIMITIVES

The field of developmental psychology has been struggling with the issue of innateness for some time (e.g., Carey, 2009; Chomsky, 1987; Elman et al., 1996; Fodor, 1975, 1981; Quine, 1960; Pinker, 1984, 1989; Spelke, 1994; Spelke, Breinlinger, Macomber, & Jacobson, 1992; Smith, 2001). Discussions often start with philosophers such as Plato, Berkeley, Descartes, Locke, Quine, Fodor, Chomsky, and others, and when moving into the subfield of cognitive development, Piaget (e.g., Piaget, 1954). The challenge is to accurately characterize the initial state of a human infant—What does the infant know? What is he or she capable of thinking and learning? What sorts of building blocks are in place at birth or shortly after? The importance of this question cannot be overstated without a characterization of the initial state, it would be difficult, if not impossible, to understand the nature of learning and developmental change.

Piaget's account of the starting state is in every textbook on developmental psychology. Most parents have heard of Piaget and know something about his ideas, be it about object permanence, or the child as an active learner, or accommodation and assimilation as mechanisms of learning. Although the credit is due to Charles Darwin as the first to systematically document children's behavior as the basis for theorizing about development, Piaget stands out as the first to propose an account, with a characterization of the initial state and a characterization of how development proceeds from one stage to the next. (The other, even more famous, developmental psychologist is, of course, Sigmund Freud, whose main concern was normal and abnormal personality development.) Armed with the progress developmental psychologists have made in the last several decades, there is now reasonable consensus that the Piagetian picture is not right, in fundamental ways. In particular, his characterization of the initial state and his stage theory have been called into question by many, based on a large body of empirical findings (e.g., see Carey, 2009; Carey & Spelke, 1996; Gelman & Baillargeon, 1983; Gopnik & Meltzoff, 1997; Spelke et al., 1992; Wellman & Gelman, 1992, for reviews).

One new perspective that has been developed in detail is the *core knowledge* proposal (Carey, 2009; Carey & Spelke, 1996; Spelke, 1994; Spelke et al., 1992; also sometimes called *core cognition*). In this view, human infants begin life with a set of core knowledge systems, and a handful of innate concepts are at the center of these reasoning systems—object, number, agent, causality, and space. Each core system has its roots in our evolutionary history and is shared with our primate relatives. Importantly, the fact that these core cognition systems participate in certain kinds of inferences is the signature for thinking of them as having conceptual content. Two versions of this point have been explicated in the literature: One focuses on the fact that these early representations are amodal (Spelke, 1994; Spelke et al., 1992), contrary to the Fodorian view that encapsulated perceptual modules are modality-specific (Fodor, 1981); the other focuses on the inferencial role these early representational structures participate in, that is, the richness of the inferences and types

of inferences (Carey, 2009). For example, the object concept is not limited to the visual modality; similar principles guide infants' perception of objects in the visual and tactile modalities (e.g., if two pieces move together—common fate—infants perceive them as parts of one object). Infants' number sense not only allows them to perceive approximate numerosities in vision and audition, but it also allows them to perform simple arithmetic operations, such as approximate addition and subtraction, an example of rich inference.

I have no doubt that the progress in developmental psychology warrants a serious reconsideration of the Piagetian account of development; I also share the confidence that the last several decades of infancy research has been illuminating. Here I raise a few questions about the core knowledge thesis and consider a revision.

The perceptual-conceptual distinction has been central to the debate about the initial state of a human infant. For various reasons, most developmental psychologists are more sympathetic to and more comfortable with the view that infants start life with perceptual capacities and primitives: They can perceive color, size, shape, and motion and, indeed, there are well-studied brain areas that are designated for representing these basic perceptual primitives (e.g., Elman et al. 1996; Karmiloff-Smith, 1990; Piaget, 1954). These views are in agreement with the long-standing philosophical tradition of empiricism (e.g. Locke, 1690/1975; Hume, 1748/1999) and Piaget's characterization of the initial state in terms of sensorimotor primitives.

Spelke, Baillargeon, R. Gelman, and their colleagues were among the first to challenge the Piagetian view, marshaling both theoretical arguments as well as new empirical evidence. The avalanche began with the seminal work of R. Gelman and Baillargeon (1983), Spelke (1985), and Baillargeon, Spelke, and Wasserman (1985). New methodological advances—in particular the development of the violation-of-expectation looking time paradigm—allowed researchers to ask questions of infants that could not be asked before. New studies with young infants barely 4 months of age suggest they may already represent persisting objects; even more astonishingly, these young infants may already have a set of principles that guide their reasoning about medium-sized objects, and these are the very same principles that adults use still—for example, continuity, solidity, and contact. Spelke et al. (1992) articulated the core knowledge view as an alternative to the standard Piagetian theory. They focused specifically on the centrality of these early concepts: Not only is the object concept embedded in a system of reasoning, it is also amodal—similar evidence is found in visual as well as tactile tasks (e.g., Streri & Spelke, 1988).

More recently, in an elegant, well-written, and ambitious synthesis of recent work on infant cognition and cognitive development in general, Carey (2009) argued forcefully against the Piagetian characterization, as well as that of the British empiricists, of the initial state. She marshaled evidence from a large body of literature supporting the claim that human infants are endowed with at least four "core cognition" systems: object, number, agent, and cause. Although Carey suggests that the format of representation for each of these concepts may be iconic, she also argues that these primitives are conceptual, using the criterion that these early representations play important inferential roles in larger

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conceptual structures. Within the core knowledge system of object cognition, a rich set of reasoning principles appears to be in place very early in development. Similarly within the core knowledge system of agent cognition, a rich set of reasoning principles seems to be in place by the end of the first year. Furthermore, the outputs of these core knowledge systems interact with each other to support even richer inferences in other reasoning tasks.

Two issues need further consideration, however. First, advances in vision science have taught us that perceptual inferences are ubiquitous in all aspects of perception, from luminance perception, surface perception, structure from motion, to generic viewpoint and object recognition (e.g., Feldman, 2012; Knill & Richards, 1996; Knill & Pouget, 2004; Weiss, Simoncelli, & Adelson, 2002). For example, Feldman and Singh (2006) showed that shape representations naturally arise as a form of probabilistic inference. The inferential mechanism tries to guess what best explains the observed shape given certain assumptions about the underlying generative process. Thus it seems unlikely that, by itself, playing an inferential role is enough for saying that the initial primitives have conceptual content. Principles of object perception—cohesion, solidity, and contact—are argued to be embedded in a system of reasoning that is causal-explanatory in nature. Perceptual inferences appear to be qualitatively similar—sets of perceptual variables are computed jointly to determine the output of the inferential mechanisms, and some are more causal than others.

Second, it may not be surprising that representations of objects and agents interact after all, what good would the outputs of the core knowledge systems do if they did not interact with each other to advance understanding of the world? Put another way, even if we started with lower-level primitives and somehow construct the concepts of object and agent, one may also expect that these concepts will interact with each other in meaningful ways further down the reasoning stream. Perceptual modules (e.g., motion detection, color perception) do deliver outputs that can be used for higher-level cognitive processes such as object recognition and categorization, yet most of us continue to think that these lower-level processes are perceptual.

It is an important discovery that early representations are amodal. However, the strong evidence that the number sense exists in visual, auditory, and action forms may not necessarily argue for its conceptual content. In the case of the number sense, experiments with young infants using visuospatial arrays, sequences of sounds, and sequences of actions all found that Weber's law applies, just like it does for many other magnitudes such as intensity of sounds, length, and luminance level (see Carey, 2009; Feigenson, Dehaene, & Spelke, 2004 for reviews). Thus amodal representations are not *necessarily* conceptual.

Another important reason why one might question the conclusion that the initial primitives are fully conceptual is that there exist real discontinuities between these primitives and later, more mature concepts, which can be, for the most part, lexicalized and tagged by mental symbols in language (e.g., *dog, ball, 3, left*).

Take the case of object, a well-studied concept in infancy. Carey and Xu (2001) argued that the properties we have evidence for in infancy for a concept of object could be thought of as properties of a mid-level attention system (e.g., Kahneman, Treisman, & Gibbs, 1983;

Scholl, 2001 Scholl & Pylyshyn, 1999). The initial criteria for individuation and tracking identity are spatiotemporal (see Spelke, Kestenbaum, Simons, & Wein, 1995; Xu, 1999; Xu & Carey, 1996; but see Wilcox, 2003; Wilcox & Baillargeon, 1998; see Xu, 2007, for a review), whereas our criteria for bona fide sortal concepts *dog* or *cup* (Hirsch, 1982; Macnamara, 1982; Xu, 1997, 2007) are kind-relevant criteria that are not part of a subsystem of vision that relies on path and motion information for individuation and identity. Perhaps a better term for the initial object concept is the "object sense," an analogy to the "number sense."

Similarly, when we take a closer look at the evidence for the number concept, there is strong evidence for an approximate number system-the "number sense" (Dehaene, 1997, 2011; Gallistel, 1990). This system, as has been argued by many, shares many properties of well-studied perceptual systems for discriminating duration, lightness, weight, or length—for example, Weber's law applies (Brannon, Abbott, & Lutz, 2004; Lipton & Spelke, 2003; Xu, 2003; Xu & Spelke, 2000; Xu, Spelke, & Goddard, 2005; see Feigenson, Dehaene, and Spelke, 2004, for a review and studies with preschoolers and adults). For example, 6-month-old infants, and even newborns, can tell the difference between 6 and 18, 8 and 16, but not 8 and 12; 9- to 10-month-old infants can tell the difference between 8 and 12 but not 8 and 10. In other words, our initial numerical representations are approximate and nondiscrete, whereas the numerical concepts we need for learning the meaning of number words (1, 2, 3, 4, 5, ...) must be symbolic and discrete. Even more importantly, the "number sense" is nothing like the number line in its formal properties: For the number sense, the difference between 8 and 16 is not equal to the difference between, say, 16 and 2.4; in fact, the difference between 8 and 16 (1:2) is the same as that between 16 and 32 (1:2). To acquire the concept of positive integer, learners would have to abandon every belief (implicitly represented in the system for numerical reasoning) they had about how numbers work and construct a brand new set of beliefs, such as how the successor function works, and how sets relate to each other and to the counting routine (Carey, 2009).

When it comes to learning the meanings of number words (1, 2, 3, 4, 5, ...), there is little dispute that genuine conceptual change is needed. Wynn (1990, 1992), Sarnecka (2008), Sarnecka & Lee (2009), and Le Corre and Carey (2007), among others, documented the developmental time course of learning to meaning of positive integers. These researchers discovered that children engage in the counting routine quite early (a practice that is strongly encouraged in the average middle-class American households), but it takes children about a year and a half to figure out how counting works in terms of the cardinality principle—children begin by figuring out the meaning of 1, then the meaning of 2 after a few months, then the meaning of 3 after another few months, and so on. Eventually they make the inductive leap that the next number on the count list represents "one more" from the one before, that is, the successor function.

A number of researchers have provided theoretical accounts of this developmental process. Carey (2009) argues that the transition from prelinguistic representations of numerosity to using symbols to think about numbers as referring to discrete quantities requires genuine conceptual change. I agree—the prelinguistics systems of approximate

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number representations and parallel individuation are not up for the job of supporting the learning of verbal counting. New representational resources need to be *constructed*.

What about causality, agency, and space? Perhaps parallel arguments can be made. There exist arguments in the literature on how causality changes from perceptual (i.e., defined by spatiotemporal parameters, Michotte) to conceptual (see the interventionist account of causality, e.g., Gopnik et al., 2004; Gopnik & Wellman, 2012; Woodward, 2003), and causal language may play an important role (e.g., Bonawitz et al., 2010). There also exist arguments on how our representations of space change drastically when spatial language is acquired (e.g., Hermer-Vazquez, Moffet & Munkholm, 2001; Hermer & Spelke, 1996; Spelke, 2003) and the initial representations are modular and encapsulated in a perceptual input analyzer. I am not aware of parallel arguments for the development of our concept of agent. My reading of the literature on the early understanding of agency is that it is rather messy and controversial—some have argued for strong nativist views (e.g., Gergely & Csibra, 2003; Gergely, Nádasdy, Csibra, & Bíró, 1995), while others favored a more piecemeal learning characterization (e.g., Meltzoff & Brooks, 2001; Woodward, 1998). One aspect of theory of mind development, false belief understanding, may give us another relevant example here—infants may have some implicit understanding of other peoples' beliefs, but these do not appear to be accessible to verbal predictions until later in development (for papers on this controversy, see Onishi & Baillargeon, 2005; Ruffman & Perner, 2005; Wimmer & Perner, 1983; and many others).

An alternative construal is that infants' initial knowledge is embedded in a set of perceptual analyzers that are capable of very sophisticated inferences and computations. Perhaps these representations are best characterized as *proto-conceptual* primitives because they do not deliver representations in the relevant format for the learning of language, and children seem to have a great deal of difficulty building other, more complex and explicit concepts out of these primitives. In other words, it is not a simple mapping problem to figure out how words refer to these concepts, and it is not a simple "tweaking" of the primitives that would do the trick. The changes that take place during and beyond infancy require new primitives, new hypotheses, and new ways of learning and thinking.

THREE TYPES OF MECHANISMS OF LEARNING AND DEVELOPMENTAL CHANGE

What are the learning mechanisms that bring about developmental change? In other words, what is "rational" and "constructivist" about the young human learner's mind and the course of development? I suggest that there are at least three types of mechanisms for learning and developmental change: (1) language as a medium for providing placeholder structures and imposing constraints on representational format; (2) Bayesian inductive learning as a way of rationally combining evidence and prior knowledge for the purpose of belief revision; cumulative belief revisions may lead to conceptual change such that peripheral concepts may become core concepts, and vice versa; and (3) explanation, analogy, thought experiments, and other internal processes of "learning by thinking" as tools

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for going beyond data and evidence to build larger conceptual structures. To date, we have some evidence for each of these three types of learning in the domains that I have considered in the first part of this paper.

Language Learning as a Vehicle for Conceptual Change

If infants' initial knowledge is not in the right format for language learning, and infants begin to learn the meaning-bearing parts of language in earnest toward the end of the first year, some developmental changes need to take place. One hypothesis that has been argued for in the literature is the idea that various parts of language provide "placeholder structures" for conceptual development (e.g., Carey, 2009; Gelman, 2003; Xu, 2002, 2007; among others). Two case studies come to mind.

In research on whether infants' representations of objects include the crucial ingredients of sortal-kind representations, my colleagues and I have suggested that initially infants individuate objects and track their identity by applying spatiotemporal criteria (e.g., since no object can travel from point A to point B without traversing a connected path in between, if that appears to happen, it means that there are in fact two numerically distinct objects in the event). Quite a lot of empirical evidence seems to support this conclusion (e.g., Spelke et al., 1995; Xu & Carey, 1996; Wynn, 1992, among others). Later on, we have suggested, infants begin to learn words for object kinds such as BALL, CUP, and DOG, and it is through word learning that infants build new concepts that allow them to individuate and track identity under sortal-kinds BALL, CUP, and DOG (see Xu, 2007, for a review). The words—count nouns that refer to sortal-kinds—impose certain constraints on the format of the representations: Symbolic (often) refers to mutually exclusive categories that reflect distinct underlying essences or causal structures, and support inferences at the level of kinds.

The case of number is similar, as sketched out earlier in the chapter. The prelinguistic representations, especially the part that is genuinely a system capable of numerical computations, the approximate number system, does not seem to play a role in acquiring the meanings of number words. Instead, the counting list provides a placeholder structure (Carey, 2009; Sarnecka & Carey, 2008) that imposes a set of constraints on the format of the representations: symbolic, discrete, generated by the successor function, goes on to infinity, among others.

Core knowledge systems may rely on language to *become* truly symbolic and discrete, and this is a *deep* conceptual change that requires new representational resources be constructed. The "language of thought" may not have existed before these changes have taken place, since by definition, the LOT is supposed to respect the syntactic constraints of a natural language (e.g., Fodor, 1975). Note that this is not a Whorfian idea—all languages have a set of syntactic tools and these are, as far as I know, universal. Even if different languages express some ideas differently using different syntactic devices, the *totality* of the thoughts we can entertain would remain the same for all human learners. A child learning English and a child learning Chinese may use different syntactic tools to transform core knowledge representations into symbolic

and conceptual representations, but at the end of the day, the thoughts that can be expressed in each language would be the same.

Bayesian Inductive Learning as a Tool for Belief Revision

Much recent research on cognitive development focuses on understanding statistical learning and probabilistic inference mechanisms (see Gopnik & Wellman 2012; Xu & Kushnir, 2012, 2013 for reviews). This line of inquiry is largely inspired by the surge of Bayesian computational models in cognitive science (see Tenenbaum, Kemp, Griffiths, & Goodman. 2011, and Griffiths, Chater, Kemp, Perfors, & Tenenbaum, 2010 for reviews).

Much of this work was motivated by the idea that we need an approach to cognitive development that is neither extreme empiricism nor extreme nativism. Nativist theories have focused primarily on specifying innate concepts and core knowledge systems, and how abstract, symbolic representations underpin not only our mature conceptual system but also that of infants and young children (e.g., Chomsky, 1987; Fodor, 1975; Pinker, 1984; Spelke, 1994), whereas empiricist theories have focused on specifying associative learning mechanisms and the graded nature of our learning and representations (e.g., Elman et al., 1996; Karmiloff-Smith, 1992; Smith, 2001). Neither view appears to provide an adequate account of all the empirical findings on cognitive and language development. The inadequacy of both extreme nativist and extreme empiricist views has led researchers to try to find a substantive middle ground (e.g., Johnson, 2010; Newcombe, Ratliff, Shallcross, & Twyman, 2010).

The rational constructivist approach blends elements of a constructivist account of development with the account of learning as rational statistical inference that underlies probabilistic models of cognition (Chater & Oaksford, 2008; Griffiths et al., 2010; Tenenbaum et al., 2011). Several basic tenets have been laid out elsewhere (see Xu & Kushnir, 2012):

- Human learning is best described as a form of rational Bayesian inference: The learner starts with some prior probability distribution over a set of hypotheses and computes the posterior probabilities of these hypotheses given the strength of the evidence as given by Bayes rule. This is a computational level characterization; that is, it describes the inferential process without making a priori commitments to how that process is instantiated at the algorithmic level (what steps to follow when a learner wants to solve a particular task) (Marr, 1982).
- Hypotheses can be represented as probability distributions. Inferences are probabilistic and graded, so hypotheses are not simply ruled in or out. Instead, learners may be more or less confident about the various hypotheses.
- Learners represent the world not just by forming associations and correlations but by constructing abstract, causal, generative models.

- Learners acquire new concepts and biases in the course of development; the newly acquired knowledge becomes part of the prior and thus constrains subsequent learning.
- Domain-general learning mechanisms may give rise to domain-specific knowledge.
- Representations may differ in their strengths; some support predictions, actions, and explanations, while others may not.
- Learners are actively engaged in the learning process, from infancy to adulthood.

Empirical evidence for this view has been rapidly accumulating in the last decade, starting with the work on causal learning in young children (Gopnik & Sobel, 2000; Gopnik et al., 2004; Gopnik & Wellman, 2012). Many studies have demonstrated that young children are sensitive to probabilistic evidence when inferring causal structure, and children and adults' behavior can be well captured by Bayes net representations and Bayesian learning models (e.g., Schulz, Bonawitz, & Griffiths, 2007; Sobel, Tenenbaum, & Gopnik, 2004).

Many domains have been investigated using this set of analytic and computational tools. In word learning, studies have shown that children are sensitive to "suspicious coincidences" and their inferences in learning the meaning of new words can be modeled by a Bayesian model that rationally combines the input data with a structured hypothesis space (e.g., Frank & Goodman, 2012; Frank, Goldwater, Griffiths, & Tenenbaum, 2010; Xu & Tenenbaum, 2007a, 2007b). In social cognition, various researchers have demonstrated that learning about preferences and theory of mind can be construed as a rational inferential process (e.g., Baker, Saxe, & Tenenbaum, 2009; Kushnir, Xu, & Wellman, 2010; Lucas, Griffiths, Xu, & Fawcett, 2009; Ma & Xu, 2011). In physical reasoning, several studies have shown that even prelinguistic infants can carry out simple probabilistic inference tasks (e.g., Denison & Xu, 2010a, 2010b; Teglas, Girotto, Gonzalez, & Bonatti, 2007; Teglas et al., 2011; Xu & Denison, 2009; Xu & Garcia, 2008; see Denison & Xu, 2012, for a review). In pedagogical learning, formal Bayesian models have led to some innovative empirical studies with children and adults (e.g., Bonawitz et al., 2011; Shafto & Goodman, 2008; Shafto, Kemp, Mansinghka, & Tenenbaum, 2011; see Shafto, Goodman, & Frank, 2012 for a review). In speech segmentation and rule learning in language, many have provided strong empirical evidence for rational inferential processes in infants (e.g., Dawson & Gerken, 2009, 2011; Frank & Tenenbaum, 2011; Gerken, 2006, 2010; Goldwater, Griffiths & Johnson, 2011). Research has also uncovered mechanisms for making inferences at multiple levels that give rise to learned inductive biases, in domains such as word learning and causal reasoning (Dewar & Xu, 2010; Lucas & Griffiths, 2010; Samuelson & Smith, 2003; Sim, Yuan & Xu, 2011; Sim & Xu, in press; Smith, Jones, Landau, Gershkoff-Stowe, & Samuelson, 2002).

Explanation, Analogy, and Thought Experiments

The rapidly growing research on statistical learning and statistical inference paints a picture of a child as a superb data-crunching machine, but not all learning is data driven.

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Some psychologists have used the phrase "learning by thinking" to refer to a set of cognitive activities that (seemingly) generate "knowledge from nowhere" (T. Lombrozo, personal communication). The basic idea is that our naïve picture of learning—that we are exposed to various learnable facts in the world—is inadequate, and many celebrated examples in the history of science show that scientists such as Galileo or Einstein arrived at major scientific breakthroughs without the benefits of grants, graduate students, and laboratories. Their scientific insights came from "mere thinking"—ways to organize and extend what we already know by manipulating existing representations and data structures. Several such cognitive activities have been demonstrated in lay people and scientists: explanation, analogy, mental simulation, and thought experiments.

One well-studied case is the self-explanation effect (Chi, Bassok, Lewis, Reimann, & Glaser, 1989; Chi, de Leeuw, Chiu, & LaVancher, 1994). In a typical experiment, some participants are told to explain to themselves when given some math problems, while others are asked to "think aloud" with the same set of math problems. The main finding is that the explainers out-performed their nonexplaining counterparts, and the explainers did better on transfer problems that went beyond the studied examples. This appears to be a clear case of generating new insights by manipulating existing data, and the mental activity of explanation plays a crucial role. More recently, Williams and Lombrozo (2010) found that in a category learning task, participants who were asked to explain were more likely to discover broad regularities that provide an account of category structure, compared to participants who were given free study time or were instructed to think aloud during the study. Their idea is that explanation may generate new knowledge by encouraging learners to find underlying rules and regularities.

Work on analogy (most notably Christie & Gentner, 2010; Gentner, 1983; Holyoak, 2012) has suggested that structural alignment is the mechanism by which a base domain is mapped onto a target domain. Such an alignment encourages the learner to see the structural similarities between two domains and allow them to use their existing knowledge in one content domain to understand the structure of a new content domain.

Other forms of "learning by thinking" include mental simulations and thought experiments. Some research suggests that we solve mechanical problems by mentally simulating the process—appealing to these visuospatial representations provides solutions that we do not seem to have access to by verbally reasoning through the problem (e.g., Hegarty, 2004). Thought experiments have been mostly studied in philosophy of science (e.g., Gendler, 1998, 2000). One celebrated example is how Galileo worked out that all objects, regardless of their weight, would fall at the same speed (contrary to the then standard Aristotelian theory).

These learning processes are not driven by new data and new evidence. Instead, the learner possesses the ability to manipulate existing representations and data structures in the head, and new insights emerge. There exists some research on how children use explanation and analogy in learning (e.g., Christie & Gentner, 2010; Legare, 2012; Legare, Gelman, & Wellman, 2010); much more work is needed to truly understand how these processes play a role in cognitive development.

A Rational Constructivist Approach to Cognitive Development WHERE DO NEW CONCEPTS COME FROM AND IS LEARNING ALL ABOUT HYPOTHESIS TESTING?

Each of the three types of learning mechanisms may give rise to new concepts. Language, especially in the form of new lexical items (i.e., words), may provide the child with the first clue that a new concept needs to be constructed. Words themselves do not provide the contents of the new concepts (e.g., repeating the word *red* a thousand times to a blind man is not going to get him to perceive redness—I thank Lila Gleitman for this example), yet words let learners create new mental symbols about which they can acquire new beliefs. It is a thorny issue whether at least some beliefs are constitutive of concepts; personally I think the answer is yes—I may be mistaken that there are only 10,000 cows living in California (a mere belief) but I would have a different concept of *cow* from you if I didn't think that cows were animals. The process of acquiring new concepts may at least be partially dependent on acquiring the relevant core beliefs. I submit that it is a real challenge to distinguish the core beliefs from the peripheral ones (e.g., Block, 1986).

Bayesian inference mechanisms are used widely for belief revision, from low-level vision to high-level language processing and learning. As beliefs are revised and small changes become large ones, some concepts will become more central to reasoning than others. As development progresses, less important concepts may become more important, and vice versa. This process may bring about genuine conceptual change of one kind: Given a network of interrelated concepts, how central a particular concept is may change over time.

Lastly, thought processes such as explanation and analogy may allow us to see new connections that we had not been aware of before. Analogy lets us impose the structure of one domain onto another domain, trying to fit all the pieces together in a new way. This is one kind of conceptual change. Explanation lets us consider what the underlying causes may be, and allows us to ignore outlier data in favor of a more coherent overall picture. Similarly, thought experiments make full use of our inductive reasoning prowess and let us imagine scenarios that are not physically possible (e.g., traveling at the speed of light).

Although some have suggested that all learning is hypothesis testing (e.g., Fodor, 1981), various developmental phenomena provide ample counter-evidence. If intuitive theories are at the foundation of conceptual development and children progress from Theory 1 to Theory 2—theories that have distinct causally central concepts—much like scientists, then we need an account of conceptual change that goes beyond mere hypothesis testing (see Carey, 1985, 2009; Gopnik & Meltzoff, 1997, for reviews and explications).

CAVEATS AND OPEN QUESTIONS

Some caveats are in order and many open questions remain. First, although it is very important to characterize the initial state, understanding learning in Bayesian terms does not depend on it. This is because as long as we can specify the prior knowledge and biases—learned or

innate-that the learner comes to a task with, we can proceed to study how that person combines prior knowledge with new evidence to choose among a set of hypotheses and to update his or her beliefs. I think this is an important point and a "bonus" for thinking about inductive learning as Bayesian updating. Second, most Bayesian models have focused on modeling the data-driven processes. It is yet to be seen if these formal models can capture the effects of language learning and "learning by thinking" (see Piantadosi, Goodman, & Tenenbaum, 2012, and Ullman, Goodman, & Tenenbaum, 2012 for some examples). The formal tools may become very useful when we want our assumptions and algorithms clearly specified. Here I discuss Bayesian inductive learning as one type of learning mechanism by focusing on the conceptual idea of hypothesis-testing and belief-revision. Third, it may turn out to be the case that the mechanisms of learning discussed here would account for the development of some aspects of core knowledge, but the jury is still out. So far we have not shown that any particular aspect of core knowledge is learned, but the developmental changes we see during the first year of life suggest that a learning story (as opposed to mere maturational one) is possible. Fourth, many have asked the question, "Where do hypotheses come from?" when we talk about Bayesian learning, and none of us has a clear answer at the moment. Here are two possibilities: Proto-conceptual primitives may provide an initial hypothesis space, and non-data-driven cognitive activities may also generate new hypotheses for the learner along the way. Lastly, most developmental psychologists have focused on demonstrating that infants and children learn quickly-within the time limits of a single, 15- to 20-minute, lab visit. However, long-term development and conceptual change are surely more complicated. Not only do we keep track of statistical evidence over time and across subdomains, we may also need the non-data-driven cognitive activities such as explanation, analogy, mental simulation, and thought experiments in order to achieve genuine, qualitative conceptual change.

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